Written Exam at the Department of Economics
Winter 2017–18

Advanced Microeconometrics

Final Exam

December 19$^{\text{th}}$ (9:00–12:00)
(3-hour closed book exam)

This exam question consists of 5 pages in total (including this cover page).

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Please note that you should answer all questions.

Indicative weighting:
Problem 1: 50%, Problem 2: 30%, Problem 3: 20%.

The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an overall assessment of the quality of the answers to the exam questions in their totality.

Problem 1

To investigate the determinants of female labor force participation, you are provided with an independent sample of $N = 2,477$ married women from the US National Longitudinal Survey of Youth (NLSY).

The dependent variable $y \equiv$ `WEEKS` measures the number of weeks worked by these women in 1990. Approximately 45% of the sample reports working full-time ($y = 52$) and 17% reports being out of the labor force ($y = 0$). Part-time workers report a number of hours that can be considered to be continuous.

The following explanatory variables are available:

- `AFQT`: A measure of cognitive ability (Armed Forces Qualifying Test, standardized test score with zero mean and unit variance).
- `EDUC`: Number of years of schooling completed.
- `HUSBINC`: Husband's income in the previous year (in thousand USD).
- `KIDS`: Binary indicator for having children ($= 1$ if at least one child, 0 otherwise).

These covariates are stored in a $N \times 5$ matrix X, where the first column is a vector of 1s for the intercept term of the model. The column vector x_i is used to denote the i^{th} row of X.

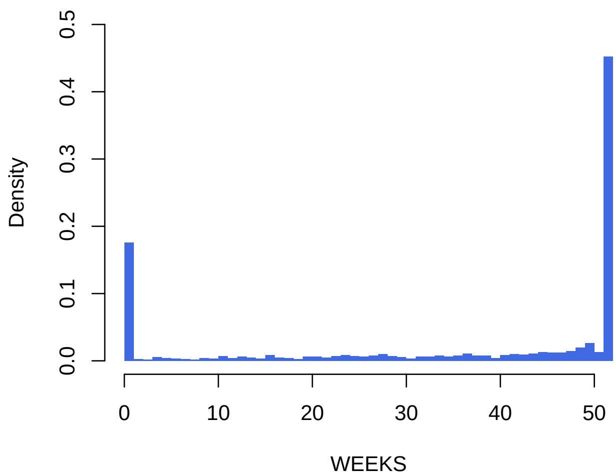

Figure 1.1: Histogram of the dependent variable `WEEKS`.

Question 1.1: Describe the features of the dependent variable `WEEKS`, and explain how the Tobit model can be extended to accommodate them. You should state your model analytically as precisely as possible.

[Note: If you do not manage to answer this question, use the Tobit model instead for this question and the remaining ones.]

Question 1.2: Derive the likelihood function $L_N(\theta; y, X)$ of the model specified in Question 1.1, where θ is the vector of model parameters.

Question 1.3: Discuss the identification of the model, and especially, whether any parameter restrictions are required for identification.

Question 1.4: Derive analytically the marginal effect of a given (continuous) explanatory variable x_j on the probability of working (either part-time or full-time). You are not asked to compute this marginal effect.

Question 1.5: Discuss the maximum likelihood estimation results presented in Table 1.1, which shows the parameter estimates (Est.) and their standard errors (SE).

Table 1.1: Maximum likelihood estimation results.

	Est.	SE
`CONST`	49.97	7.85
`AFQT`	0.14	0.05
`HUSBINC`	-0.24	0.05
`KIDS`	-31.19	2.67
`EDUC`	1.56	0.62
σ	44.51	1.23

Problem 2

Consider an *iid* sample $y = (y_1, \ldots, y_N)'$ drawn from an exponential distribution with scale parameter $\theta > 0$ such that, for each $i = 1, \ldots, N$:

$$y_i \overset{iid}{\sim} \mathcal{E}xpon(\theta), \qquad f(y_i \mid \theta) = \frac{1}{\theta} \exp\left\{-\frac{y_i}{\theta}\right\}, \qquad \mathrm{E}[y_i] = \theta.$$

Question 2.1: Propose a *natural conjugate* prior distribution for θ. Derive the corresponding posterior distribution.

You may use one of the distributions of Table 2.1. Justify your choice.

Question 2.2: Show that Jeffreys' prior for this model is $p(\theta) \propto \theta^{-1}$. Derive the corresponding posterior distribution of θ.

Hint: Remember that Jeffreys' prior is proportional to the square root of the determinant of the information matrix, i.e., $p(\theta) \propto |\mathcal{I}(\theta)|^{1/2}$, which simplifies to $p(\theta) \propto \mathcal{I}(\theta)^{1/2}$ in the present scalar case.

Table 2.1: Some probability distributions.

Distribution	Density $f(\theta \mid a, b)$	Mean
Uniform	$\dfrac{1}{b-a}$	$\dfrac{a+b}{2}$
Beta	$\dfrac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)}$	$\dfrac{a}{a+b}$
Gamma	$\dfrac{1}{\Gamma(a)b^a}\theta^{a-1}\exp\left\{-\dfrac{\theta}{b}\right\}$	ab
Inverse-Gamma	$\dfrac{b^a}{\Gamma(a)}\theta^{-a-1}\exp\left\{-\dfrac{b}{\theta}\right\}$	$\dfrac{b}{a-1}$ (for $a > 1$)

Problem 3

Consider the following MATLAB code:

```
% Inputs:
%      y     vector  Nx1
%      x     vector  Nx1
%      x0    scalar  1x1
%      h     scalar  1x1
function [yhat] = regress(y,x,x0,h)
    kern = @(z) normpdf(z);
    w = kern((x-x0)./h);
    w = w./sum(w);
    yhat = sum(w.*y);
end
```

Question 3.1: Express in mathematical terms what this MATLAB function computes. You should only provide one or two equations to answer this question. Be explicit about the notation.

Question 3.2: Using only words *(no equations required)*, explain *briefly* the methodology implemented by this function. In particular, describe the role of the two scalars “x0” and “h”, and explain the values they can take.